Math 4 Honors
Implicit Differentiation Madness!

Name _____
Date _____

For each of the following equations, find $\frac{dy}{dx}$ using **implicit differentiation**.

1. $x^2 - 5xy + 3y^2 = 7$

$$\frac{d}{dx}\left(x^2 - 5xy + 3y^2\right) = (7)\frac{d}{dx}$$

$$\begin{aligned} 2x - (5x\frac{dy}{dx} + 5y) + 6y\frac{dy}{dx} &= 0 \\ 2x - 5x\frac{dy}{dx} - 5y + 6y\frac{dy}{dx} &= 0 \\ \frac{dy}{dx}(-5x + 6y) &= -2x + 5y \\ \frac{dy}{dx} &= \frac{-2x + 5y}{-5x + 6y} \\ \frac{dy}{dx} &= \frac{2x - 5y}{5x - 6y} \end{aligned}$$

2. $4x^2 - 9y^2 = 17$

$$\frac{d}{dx}\left(4x^2 - 9y^2\right) = (17)\frac{d}{dx}$$

$$\begin{aligned} 8x - 18y\frac{dy}{dx} &= 0 \\ -18y\frac{dy}{dx} &= -8x \\ \frac{dy}{dx} &= \frac{8x}{18y} \\ \frac{dy}{dx} &= \frac{4x}{9y} \end{aligned}$$

3. Find the slope of $xy^2 + x = 1$ at $\left(\frac{1}{2}, 1\right)$

$$x y^2 = 1 - x$$

$$x \cdot 2y\frac{dy}{dx} + y^2 = -1$$

$$\frac{dy}{dx} = \frac{-1 - y^2}{2xy}$$

$$\left.\frac{dy}{dx}\right|_{\left(\frac{1}{2},1\right)} = \frac{-1 - (1)^2}{2\left(\frac{1}{2}\right)(1)}$$

$$= \frac{-2}{1}$$

$$= -2$$

Challenge:

4. $\cos(x+y)+\sin(x+y)=\frac{1}{3}$

$$D(\cos(x+y))+D(\sin(x+y))=D(\frac{1}{3})$$

$$\begin{aligned}
-\sin(x+y)(1+\frac{dy}{dx})+\cos(x+y)(1+\frac{dy}{dx}) &= 0\\
-\sin(x+y)-\sin(x+y)(\frac{dy}{dx})+\cos(x+y)+\cos(x+y)(\frac{dy}{dx}) &= 0\\
(\frac{dy}{dx}(\cos(x+y)-\sin(x+y)) &= \sin(x+y)-\cos(x+y)\\
\frac{dy}{dx} &= \frac{\sin(x+y)-\cos(x+y)}{\cos(x+y)-\sin(x+y)}\\
\frac{dy}{dx} &= -1
\end{aligned}$$

5. If $y^3 + y = x^2$, then $\frac{dy}{dx} =$

(A) 0 (B) $\frac{x}{2}$ (C) $\frac{2x}{3y^2}$ (D) $2x - 3y^2$ (E) $\frac{2x}{1+3y^2}$

$$\frac{d}{dx}(y^3 + y) = \frac{d}{dx}(x^2)$$

$$3y^2\frac{dy}{dx} + \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx}(3y^2+1) = 2x \quad \rightarrow \quad \frac{dy}{dx} = \frac{2x}{1+3y^2}$$

6. If $x^2 + xy = 10$, then when $x = 2$, $\frac{dy}{dx} =$

$2^2 + 2y = 10$, $2y = 6$, $y = 3$, $(2, 3)$

(A) $-\frac{7}{2}$ (B) -2 (C) $\frac{7}{2}$ (D) $\frac{3}{2}$

$$\frac{d}{dx}(x^2 + xy) = (10)\frac{d}{dx}$$

$$2x + y + x\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} = -2x - y$$

$$\frac{dy}{dx} = \frac{-2x-y}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{-2(2)-3}{2} = -\frac{7}{2}$$

7. If $(x + 2y)\cdot\frac{dy}{dx} = 2x - y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?

(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$

$$\frac{dy}{dx} = \frac{2x-y}{x+2y}$$

$$\left.\frac{dy}{dx}\right|_{(3,0)} = \frac{2(3)-0}{3+2(0)} = \frac{6}{3} = 2$$

$$\frac{d^2y}{dx^2} = \frac{(x+2y)\left(2-\frac{dy}{dx}\right) - (2x-y)\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$$

$$= \frac{(3+2\cdot0)(2-2) - (2\cdot3-0)(1+2\cdot2)}{(3+2\cdot0)^2}$$

$$= \frac{0 - 6(5)}{9} = -\frac{30}{9} = -\frac{10}{3}$$